OKLAHOMA SCHOOL of SCIENCE and MATHEMATICS

17th Annual OSSM Middle School Math:

An Awesome Contest—Round Two

7th and 8th Grade Test

Spring 2019

Before you begin:

1. Please write your name on your answer sheet.
2. On your name tag you have been given a three-digit OSSM student id number. On the answer sheet, in the lower right-hand corner, there is a place to fill in the three-digit student id number (use the first three boxes). Fill in the appropriate bubbles for your OSSM student id.

Directions: Use the scratch paper provided to do your work. Calculators are needed. Choose the appropriate answer, and then fill in the corresponding bubble ON THE ANSWER SHEET. DO NOT MAKE ANY STRAY MARKS ON YOUR ANSWER SHEET; it may cause the machine to misread your answer sheet and may disqualify your score.

This is a 40-question, 1-hour contest. All questions are multiple-choice. Figures are not to scale. Each question is worth one point. Your score will be the number of correct answers. There is no partial credit or penalty for wrong answers. Please continue working or reworking problems until time is called.

There is a Prize Question on the back page.
Write your answer ON THE BACK OF THE ANSWER SHEET.
All students should answer the Prize Question.

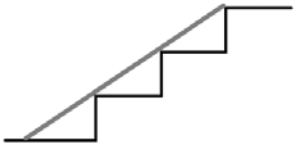
Do Not Open or Turn Over Until Instructed To Do So

Fill in your answers ON THE ANSWER SHEET by filling in the corresponding bubble.

DO NOT MAKE ANY STRAY MARKS ON YOUR ANSWER SHEET!

You may use this space for scratch paper

WRITE YOUR ANSWERS ON THE ANSWER SHEET. DO NOT MAKE ANY STRAY MARKS ON YOUR ANSWER SHEET

1. If $x + 8 = 0$, and $\sqrt{y} = 4$, then what is the value of $\frac{2x+y}{y}$? a. -16 b. -7 c. 0 d. 2 e. 16	1. a. b. (c) d. e.
2. A triangle has an area of 25 and a height of 10. What is the width of the base? a. 0.8 b. 1 c. 1.25 d. 5 e. 10	2. a. b. c. (d) e.
3. If Charles Darwin brings a dozen cookies to a party, Michael Faraday brings half a dozen cookies, and Louis Pasteur brings three cookies, but they all share equally, how many cookies do they each eat? a. 3 b. 6 c. 7 d. 9 e. 21	3. a. b. (c) d. e.
4. There are three 7-inch tall steps leading to the porch of a house as pictured. Each step is 11 inches deep. If a board is laid against the steps, what would be the slope of the board? a. $\frac{11}{21}$ b. $\frac{7}{11}$ c. $\frac{2}{3}$ d. $\frac{11}{7}$ e. $\frac{33}{7}$ 	4. a. (b) c. d. e.
5. Triangles that are *similar* have corresponding angles are equal, but the corresponding sides are proportional. Consider the triangles with side lengths given. Which triangles are similar? I. 3-4-5 II. 5-12-13 III. 9-12-15 IV. 12-16-25 V. 10-20-26 a. I & III only b. II & V only c. I, III, & IV only d. I & IV only e. I & III are similar, as are II & V	5. (a) b. c. d. e.
6. If $2A - B = -1$ and $-3A + 2B = 2$, then $B - A =$ a. -2 b. -1 c. 1 d. 2 e. 3	6. a. b. (c) d. e.
7. Which of the following is equivalent to $\frac{15x^{-4}y^{11}z^3}{5x^{-3}y^7z}$? a. $\frac{3y^4z^2}{x}$ b. $\frac{10y^4z^2}{x}$ c. $\frac{3x^{-1}y^4}{z^{-3}}$ d. $\frac{10y^4z}{x^{-7}}$ e. $3x^{-1}y^4z$	7. (a) b. c. d. e.
8. If $\sqrt{x} = y$ and $\sqrt{y} = z$ and $\sqrt{z} = 16$, then $x =$ a. 16^3 b. 16^4 c. 16^6 d. 16^8 e. 16^{16}	8. a. b. c. (d) e.
9. Albert Einstein had 2^{17} hairs on his head. If he lost 100 hairs every day for 30 days after, but none grow back, which of these is closest to the number of hairs he had left? a. 3^{11} b. 5^7 c. 6^6 d. 2^{16} e. 7^6	9. a. b. c. d. (e)
10. If $y = 1 - \frac{1}{\frac{n}{m}+1}$, which of the following is equal to $\frac{1}{y}$? a. $\frac{m+n}{n-2m}$ b. $\frac{m+n}{n}$ c. $\frac{n}{m}$ d. $\frac{nm+m}{1-m}$ e. $\frac{1-m}{n+m}$	10. a. (b) c. d. e.
11. Which two are lines that are perpendicular to one another? I. $y = 5x + 2$ II. $y = \frac{1}{5}x - 7$ III. $y = 10x + 2$ IV. $y = -5x - 2$ V. $y = -\frac{1}{5}x^2$ a. I & II b. I & III c. I & V d. II & IV e. III & IV	11. a. b. c. (d) e

12. A farmer is using 200 feet of fence to enclose a divided rectangular pen along her barn wall. She will not need a fence on the barn side of the enclosure. Which of these is closest to the area of the largest total enclosure she can create?

a. 2500 ft^2 b. 2800 ft^2 c. 3200 ft^2 d. 3300 ft^2 e. 5000 ft^2

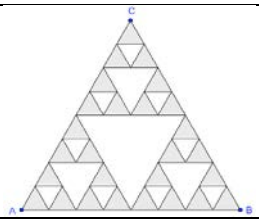
12. a. b. c. (d) e

13. On Round 2 of An Awesome Contest last year there were 180 participants. $\frac{1}{5}$ of the participants got between 0 and 9 questions correct, $\frac{2}{3}$ of them got 10 to 19 questions correct, $\frac{1}{10}$ got 20 to 29 questions correct. The rest got 30 to 40 of the questions correct. No participants are in more than one group. How many participants got 30 to 40 questions correct?

a. 0 b. 3 c. 6 d. 8 e. 9

13. a. b. (c) d. e.

14. To the nearest whole number, what percentage of the large triangle shown here is shaded?



a. 33% b. 36%
c. 40% d. 42%
e. 45%

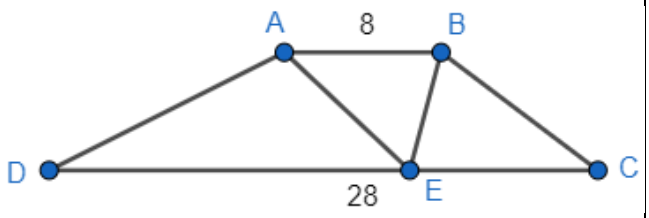
14. a. b. c. (d) e.

15. Which of the following equations is satisfied by all of the numbers -2, -1, 1, and 2?

a. $(x-2)(x+2)=0$ b. $(x+1)^2(x+2)^2=0$
c. $(x-1)^2(x-2)^2=0$ d. $(x^2-1)(x^2-2)=0$
e. $(x^2-1)(x^2-4)=0$

15. a. b. c. d. (e)

16. In the figure below, segments $\overline{AB}$ and $\overline{DC}$ are parallel and point E lies on $\overline{DC}$. If $AB=8$, $CD=28$, and the area of triangle ABE is 24, then what is the area of trapezoid $ABCD$?



a. 54 b. 72
c. 94 d. 108
e. 112

16. a. b. c. (d) e.

17. There are 365 days in 2019, and 366 days in 2020 (since it's a leap year). If January 1, 2019, is a Tuesday, then what day of the week is December 31, 2020?

a. Monday b. Wednesday c. Thursday d. Friday e. Sunday

17. a. b. (c) d. e.

18. The fraction 22/7 is sometimes used as an approximation for π. What is the 2019th digit after the decimal point when 22/7 is written out as a decimal?

a. 2 b. 4 c. 5 d. 7 e. 8

18. (a) b. c. d. e.

19. A *kite* is a quadrilateral which has two pairs of equal-length sides, and each pair of equal-length sides shares an endpoint. Which of the following is true of every possible kite?

a. Its two diagonals intersect each other at their midpoints
b. Its two diagonals are perpendicular to each other
c. Its two diagonals are congruent to each other
d. Each of its two diagonals is longer than each of its four sides
e. The sum of its angles is $180°$

19. a. (b) c. d. e

20. Three different lines are situated in a plane. Let N be the number of points in the plane where at least two of the three lines intersect. Given the different possible arrangements of the three lines, how many different values could the number N possibly have?

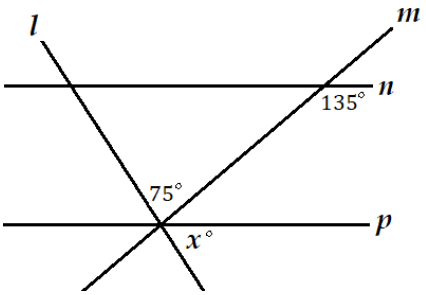
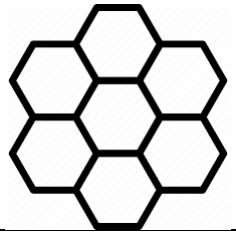
a. more than 4 b. 4 c. 3 d. 2 e. 1

20. a. (b) c. d. e.

21. If M is the largest number less than 1000 that is both a perfect square and a perfect cube, what is the sum of the digits of M?

a. 12 b. 14 c. 16 d. 18 e. 20

21. a. b. c. (d) e.

22. In the figure below (not necessarily drawn to scale), lines n and p are parallel, and lines l, m, and p meet at a point. What is the value of x? a. 45 b. 50 c. 55 d. 60 e. 65 	22. a. b. c. (d.) e.
23. You have *n* identical wooden cubes. If you set them up side by side into a long chain, glue adjacent cubes together, and then paint the outside of this new solid red, how many square faces of the original cubes did you paint in total? a. $4n+2$ b. $4n+6$ c. $5n$ d. $5n+1$ e. $5n+2$	23. (a.) b. c. d. e.
24. 20% of 80% of 20 is equal to which of the following? a. 50% of 50% of 20 b. 80% of 20% of 80 c. 20% of 20% of 80 d. 20% of 20% of 20% of 20% of 20 e. 20% of 20% of 20% of 20% of 20% of 20	24. a. b. (c.) d. e
25. How many line segments appear in the following geometric design? a. 27 b. 30 c. 33 d. 36 e. 42 	25. a. (b.) c. d. e
26. A *perfect power* is a number which can be written as a^b where a and b are positive whole numbers, and $b \geq 2$. For example, $6^2 = 36$ and $2^3 = 8$ are both perfect powers. How many two-digit perfect powers are there? a. 4 b. 5 c. 6 d. 7 e. 8	26. a. b. c. d. (e)
27. $\left\|0-\left\|1-\left\|2-\left\|3-\left\|4-5\right\|\right\|\right\|\right\|\right\| =$ a. 1 b. 2 c. 3 d. 4 e. 5	27. (a.) b. c. d. e
28. A *Sophie Germain prime* is a prime number p such that $2p+1$ is also prime. For example, the prime number 3 is a Sophie Germain prime because $2 \cdot 3 + 1 = 7$ is also prime. Which one of the numbers below is a Sophie Germain prime? a. 37 b. 39 c. 41 d. 43 e. 47	28. a. b (c.) d. e
29. On Day 1, Bernhard Riemann eats 1 pound of spaghetti. On Day 2, he eats 2 pounds of spaghetti. On Day 3, he eats 3 pounds of spaghetti, and so on. At the end of Day 3 he will have eaten 6 pounds of spaghetti, <u>total</u>. Assuming he keeps this up, on which day will he finally have eaten over 10,000 pounds of spaghetti, <u>total</u>? a. Day 141 b. Day 142 c. Day 143 d. Day 144 e. Day 145	29. (a.) b. c. d. e
30. When my hamster runs in her hamster wheel, the wheel spins at a rate of 20 revolutions per minute. If its diameter is 10 inches, then how far has my hamster effectively run after 30 seconds? a. 50 π inches b. 100 π inches c. 150 π inches d. 200 π inches e. 250 π inches	30. a. (b.) c. d. e.
31. Grace Hopper reaches into a drawer that contains 2 black socks and 2 white socks. She pulls out two socks completely at random. What is the probability that the two socks match? a. $1/6$ b. $1/5$ c. $1/4$ d. $1/3$ e. ½	31. a. b. c. (d.) e.

32. If $x > 20$ and $y < 4$, then which one of the following equations cannot be true?

a. $x + y = 24$ b. $x + y = 17$ c. $x - y = 17$ d. $xy = 80$ e. $\frac{x}{y} = 5$

32. a. b. c. d. (e)

33. Archimedes and Zeno each run at constant (but different) speeds. Archimedes gives Zeno a 10-mile head start in a long footrace. And they're off! After 1 hour, Archimedes reaches Zeno's starting point, but meanwhile Zeno has run forward 1 mile. How much **more** time will it take Archimedes to completely catch up with Zeno?

a. 1/11 of an hour b. 1/10 of an hour
c. 11/100 of an hour d. 1/9 of an hour
e. Archimedes will never catch up with Zeno no matter how long they both run

33. a. b. c. (d) e.

34. The *factorial* of n, written $n!$, is the product of all the whole numbers from n down to 1. For example 3! = 3×2×1 = 6, and 4! = 4×3×2×1 = 24. We can also repeat the factorial operation—for example, 3!! = (3×2×1)! = 6! = 6×5×4×3×2×1 = 720. Which of these numbers is the largest?

a. 65536!! b. 256!!! c. 16!!!! d. 4!!!!! e. 2!!!!!!

34. a. b. c. (d) e.

35. In my drawer I have 12 Marvel movies, 9 DC Comics movies, 6 Tolkien movies, and 9 Harry Potter movies. If I randomly take out two movies, what is the probability that it would be one Harry Potter and one Marvel movie?

a. $\frac{1}{12}$ b. $\frac{1}{4}$ c. $\frac{1}{3}$ d. $\frac{6}{35}$ e. $\frac{7}{12}$

35. a. b. c. (d) e.

36. Srinivasa Ramanujan wrote down a list of seven positive whole numbers, all different. The mean (average) of the numbers on his list is 22 and the median (middle number when arranged in numerical order) is 4. If N is the smallest possible value for the largest number on his list, what is the sum of the digits of N?

a. 9 b. 10 c. 11 d. 12 e. 13

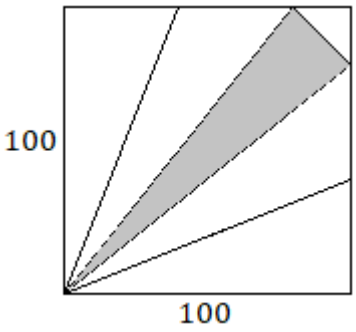
36. a. b. c. d. (e)

37. A rectangle has a side of length $2x$, and its area is $12x^2$. What is the rectangle's other side length?

a. $6x$ b. $6x^2$ c. $10x$ d. $10x^2$ e. 24x

37. (a) b. c. d. e.

38. A square with side length 100 has four segments drawn from its lower left corner in such a way that they cut the square into five regions of equal area. What is the area of the triangle formed by connecting the ends of the two middle line segments, as shown?



a. 1600 b. $1200\sqrt{2}$ c. 1750
d. 1800 e. $1350\sqrt{2}$

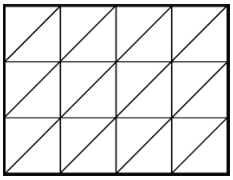
38. a. b. c. (d) e.

39. How many ping-pong balls could you place in a single layer, without stacking, on the surface of a ping-pong table?

a. about 300 b. about 3000
c. about 30,000 d. about 300,000
e. about 3,000,000

39. a. (b) c. d. e.

40. A caterpillar at the lower left corner of the rectangle below will make its way to the upper right corner. At each step it moves either upward, rightward, or diagonally up and to the right, following the lines in the diagram. How many different paths can it take to reach its destination?



a. 128 b. 129 c. 130
d. 131 e. 132

40. a. (b) c. d. e.

Prize Question

WRITE YOUR ANSWER ON THE BACK OF YOUR ANSWER SHEET

Every participant should write down any whole number of their choosing between 1 and 1000. The participant who writes down the number that is closest to $\frac{2}{3}$ of the average of all of the numbers written down by participants for this question will win a small prize.

You may use this space for scratch paper,

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