Sixteenth Annual Middle School Mathematics Contest

7th /8th Grade Test

Round Two, Spring 2018

Before you begin:
1. Please verify that the information on the sticker on your answer sheet is correct. If anything is incorrect, please mark through it and write the correct information neatly next to it.
2. On the sticker on your answer sheet you have been given a three-digit OSSM student id. On the answer sheet, there is a place to fill in the three-digit student id (use the first three boxes). Fill in the appropriate bubbles for your OSSM student id.

Directions: Use the scratch paper provided to do your work. Calculators are allowed. Choose the appropriate answer, and then fill in the corresponding bubble on the answer sheet. Figures are not to scale. DO NOT MAKE ANY STRAY MARKS ON YOUR ANSWER SHEET; it may cause the machine to misread your answer sheet and may disqualify your score.
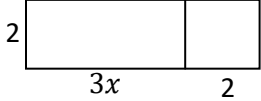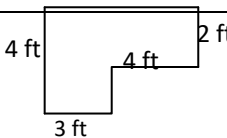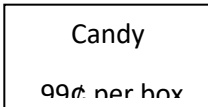
This is a 40-question, 1-hour contest. All questions are multiple-choice. Each question is worth one point. Your score will be the number of correct answers. There is no partial credit or penalty for wrong answers. Please continue working or reworking problems until time is called.

The tie-breaker question is on the back page.
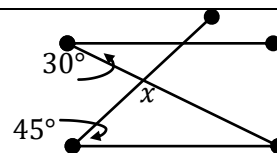Write your work and your answer on the back of the answer sheet.
The tie-breaker question will only be graded in the event that there is a tie for a trophy between two contestants that cannot be settled without the tie-breaker. The best or more complete answer among those who are tied will win the tie-breaker. A complete solution is not necessary to win a tie. Show the work for the tie-breaker on the page with the question.

Do Not Open or Turn Over Until Instructed To Do So

1. Consider the figure. Which of these represents the total area of the figure?
   a. $3x + 4$ b. $6x + 2$ c. $6x + 4$
   d. $6x + 8$ e. $12x$

2. If $A \otimes B = \dfrac{A-B}{A+B}$, evaluate $(5 \div 2) \otimes (3 \div 4)$.

   a. $\dfrac{7}{13}$ b. $\dfrac{8}{15}$ c. $\dfrac{37}{40}$ d. $\dfrac{59}{26}$ e. $\dfrac{127}{40}$

3. A rectangle measures $10cm$ by $6\ cm$. If a semi-circle with the same diameter as the length of one side will be set against that side, what is the maximum possible area of the new combined figure?

   a. $\dfrac{120+9\pi}{2}\ cm^2$ b. $60 + 10\pi\ cm^2$ c. $60 + 12\pi\ cm^2$ d. $\dfrac{120+25\pi}{2}\ cm^2$ e. $60 + 25\pi\ cm^2$

4. A pizza measures 14" in diameter, including a 1" crust. Approximately what percent of the area of the pizza is crust?
   a. 7%   b. 14%   c. 17%   d. 24%   e. 27%

5. If a 12" string is going to have beads alternating in size ½" and ¼" in diameter, allowing 1" on both ends for tying, how many beads will fit?
   a. 13   b. 14   c. 26   d. 27   e. 40

6. How many square tiles, each measuring six inches on a side, are needed to cover a floor as represented in the figure?
   a. 20   b. 40   c. 60   d. 80   e. 120

7. A special price is being offered for candy at the movie theater. George has $2. How many boxes of candy can he buy?
   a. 2   b. 2.02   c. $2.\overline{02}$ d. 202

Candy

99¢ per box

8. How many whole numbers n, with $1 \le n \le 30$, have the property that n^n is a perfect square?
   a. 15   b. 16   c. 17   d. 18   e. 19

9. Arjun ate one Thin Mint on Monday. Each day after that, he ate twice as many Thin Mints as he had eaten in all previous days put together. How many Thin Mints had he eaten, IN TOTAL, by the end of Friday (the fifth day)?
   a. 16   b. 31   c. 54   d. 63   e. 81

10. Which of the following is always equal to $(2A - 2B)^2$ for any real numbers A and B?
    a. $2A^2 - 2B^2$ b. $4A^2 - 4B^2$ c. $2(B - A)^2$ d. $4(B - A)^2$ e. $2A^2 - 4AB + 2B^2$

11. $(-1)(-1)(-1) - (-1)(-1)(-1) - (-1)(-1)(-1) - (-1)(-1)(-1) =$
    a. -4 b. -2 c. -1 d. 0 e. 2

12. Which of the following is equal to $1^2 + 2^2 + 3^2 + 4^2 + 5^2$?
    a. $3^2 + 3^2 + 3^2 + 3^2 + 3^2$ b. $(1 + 2 + 3 + 4 + 5)^2$ c. $2^1 + 2^2 + 2^3 + 2^4 + 2^5$
    d. $(1 \times 3) + (2 \times 4) + (3 \times 5) + (4 \times 6)$ e. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

13. If you earn $200, and the federal government takes 20% as income tax, then you give 10% of what's left to charity, then you donate 10% of what's left to your school, and then you pick up 8 nickels, how much money do you have?
    a. $80.40   b. $120.00   c. $120.40   d. $130.00   e $142.80

14. If $2x + 3y = 44$, and $x + y = 17$, then what is the value of $x - y$?
    a. -7 b. -3 c. 4 d. 6 e. 7

15. Which of the following gives the circumference, in meters, of a circle whose area is A square meters?

    a. $\dfrac{2\sqrt{A}}{\pi}$ b. $2\sqrt{\dfrac{A}{\pi}}$ c. $\sqrt{\dfrac{A}{2\pi}}$ d. $2\sqrt{\pi A}$ e. $2\pi\sqrt{A}$

16. Consider the figure. If the horizontal lines are parallel, what is the value of the angle x?

    a. 45°   b. 75°   c. 95°

         d. 105°   e. 110°

17. π is the ratio of a circle's circumference to its diameter. τ is defined by some as the ratio of a circle's circumference to its radius. What is the formula for the area of a circle in terms of τ and diameter instead of π and radius ?

    a. $\frac{\tau d^2}{8}$ b. $\frac{\tau d^2}{4}$ c. τd^2 d. $2\tau d^2$

18. You have 2 quarters, 1 dime, and 4 pennies in your pocket. What is the probability that you can exactly match the change part of your bill (which is a random number of cents between 0 and 99, inclusive)?
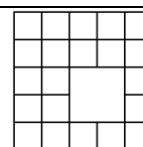
    a. $\frac{13}{50}$ b. $\frac{27}{100}$ **c. $\frac{3}{10}$**   d. $\frac{16}{25}$ e. $\frac{13}{20}$

19. If $\frac{2}{13} < \frac{1}{n} < \frac{3}{13}$, there are two possible integer solutions for n. What is the product of those two values?

    a. 20   b. 24   **c. 30**   d. 35   e. 42

20. Let N be the number of squares appearing in the figure. What is the units digit of N?

    a. 5   b. 6   c. 7   **d. 8**   e. 9

21. $2018 - 2016 + 2014 - 2012 + 2010 - 2008 + 2006 - 2004 + \cdots - 12 + 10 - 8 + 6 - 4 + 2 =$

    a. 1006   b. 1008   **c. 1010**   d. 2018   e. 2020

22. Mr. Kuehl has 168 sophomores and 180 freshmen visiting OSSM. If he wants to divide them up into mixed groups with the same number of sophomores in each group and the same number of freshmen in each group, what is the largest number of groups he may have?
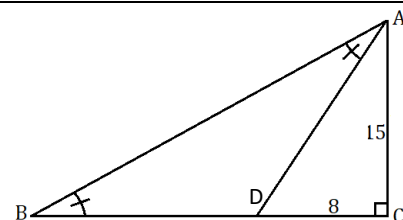
    a. 6   b. 8   c. 10   **d. 12**   e. 24

23. Every day in March, Chenwei saved a penny, except on prime-numbered days, he saved two pennies. How many pennies did he save in March? (Note: March has 31 days.)

    a. 42   b. 43   c. 44   d. 45   e. 46

24. In the diagram, $CD = 8$, $AC = 15$, $\angle C$ measures 90°, and $\angle BAD = \angle ABD$.
    What is the area of triangle ABC?

    a. 180   **b. 187.5**   c. 192

         d. 196   e. 212.5

25. How many different ways are there to rearrange the letters in OSSM, if the two S's cannot be placed next to each other?
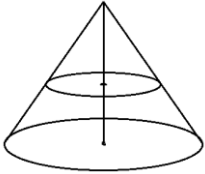
    a. 6   b. 8   c. 9   d. 10   e. 12

26. Prof. Chaves, a stamp collector, has a large number of bluejay and cardinal stamps. She is equally happy to either (1) gain two bluejay stamps and one cardinal stamp; or (2) gain three bluejay stamps and lose one cardinal stamp. Which of the following expresses the relative value she places on bluejay and cardinal stamps?

    a. 3 cardinal stamps are worth 1 bluejay stamp   **b. 2 cardinal stamps are worth 1 bluejay stamp**
         c. 1 cardinal stamp is worth 1 bluejay stamp   d. 1 cardinal stamp is worth 2 bluejay stamps
              e. 1 cardinal stamp is worth 3 bluejay stamps

27. An overachieving toddler builds a square structure with no roof or floor, and four walls, each one foot long and one foot high, using two inch cubic blocks. How many blocks were used?

    a. 120   b. 126   c. 132   d. 138   e. 144

28. It is known that Adhya is taller than Brittany, and also that Caleb is shorter than Diya, and that Diya is taller than Brittany. No two of them are the same height.
Given this information, how many different orderings of these four students, from shortest to tallest, are possible?
a. 3   b. 4   c. 5   d. 6   e. 7

29. If $x(x - 1) = 72$, then what is the difference between the largest possible, and the smallest possible values of $x(x + 1)$?
a. 34   b. 36   c. 38   d. 40   e. 42

30. If you have 96 one inch cubes arranged as a solid rectangular prism, there are twelve possible arrangements. What is the smallest surface area possible for the solid?
a. $128 \ in^2$ b. $136 \ in^2$ c. $144 \ in^2$ d. $152 \ in^2$ e. $160 \ in^2$

31. In the correctly worked addition problem here, each letter represents a different digit from 0 to 9. What is the value of $T-A$?
a. 1   b. 2   c. 3   d. 4   e.5

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  OSSM
+ MATH
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 MMHMM
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32. Remy's top running speed is 12 mph, but she can only maintain that speed for up to 10 minutes at a time. After that, she has to slow down to a 6 mph jog for 5 minutes before she can run again. If she runs at top speed as much as possible, and only switch to jogging when she has to, how long will it take her to cover a distance of 12 miles?
a. $66\frac{2}{3}$ minutes   b. 70 minutes   c. 72 minutes   d. 75 minutes   e. 80 minutes

33. What is the first time after 3:00 p.m. when the hour and minute hands of a clock are exactly on top of each other?
a.3:14 p.m.   b. 3:15 p.m.   c. 3:16 p.m.   d. 3:17 p.m.   e.   4:20 p.m.

34. How many odd numbers between 1 and 1,000 (inclusive) contain the digit 4 at least once?
a. 50   b. 95   c. 100   d. 110   e. 133

35. Place the following "power towers" in order from smallest to largest:

i. $2^{2^{2^{2}}}$ ii. $3^{3^{3^{3}}}$ iii. $4^{4^{4}}$

a. i, ii, iii   b. i, iii, ii   c. ii, iii, i   d. iii, i, ii   e. iii, ii, i

36. Peter's dataset has five values in it, and the mean is numerically equal to the square of the median. If all five values are different positive integers, what is the smallest possible value for the largest of the five numbers?
a. 19   b. 20   c. 35   d. 36   e. 45

37. If you take a right circular cone and slice it with a horizontal plane halfway up its height (see figure), what is the ratio of the volume of the new, small upper cone to the volume of the original, large cone? $V = \frac{1}{3}\pi r^2 h$
a. 1:8   b. 1:6   c. 1:4   d. 1:3   e. 1:2

38. A fish tank is 20% full of water. At some point, a plug is removed allowing water to drain out the bottom at a rate of 0.75 gallons per minute. At that same moment, a hose is turned on which pours water into the tank at a rate of 1.4 gallons per minute. Twenty minutes later, the tank starts to overflow. How large is the tank? (Round your answer to the nearest whole number of gallons.)
a. 10 gallons   b. 12 gallons   c. 13 gallons   d. 16 gallons   e. 17 gallons

39. One of the answer choices below is the square of a prime number. Which one?
a. 488055749064197157   b. 488055749064197161   c. 488055749064197163
d. 488055749064197165   e. 488055749064197168

40. If $x^2 + y^2 = 400$, what is the smallest positive whole number that CANNOT be equal to $x + y$?
a. 25   b. 26   c. 27   d. 28   e. 29

Tie-Breaker

Write your answer and your work <u>on the back of the answer sheet</u>.

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A poem's rhyme scheme is a way of using a sequence of letters to classify which lines rhyme with each other. For example, the poem

Roses are red
Violets are blue
This is a poem
I made up for you

has an ABCB rhyme scheme, since only the 2nd and 4th lines rhyme with each other.

In general, an n-line rhyme scheme always starts with the letter A and then either repeats a previously used letter (if the current line rhymes with one or more previous lines) or introduces the next unused letter of the alphabet (if the current line doesn't rhyme with any previous lines). A 4-line poem without any rhymes would have the scheme ABCD. If all four lines rhymed with each other, it would be AAAA.

How many 7-line rhyme schemes are there? Show your work or explain your reasoning on the back of the answer sheet.