Oklahoma School of Science and Mathematics
Fourth Annual Middle School Mathematics Contest
Round One, Spring, 2006

Explanation of solutions

1. $\frac{12 \text{ eggs}}{\text{carton}} \times \frac{8 \text{ cartons}}{\text{crate}} \times \frac{9 \text{ crates}}{\text{container}} = \frac{864 \text{ eggs}}{\text{container}}$

2. $200^6 = (2\times 100)^6 = 2^6 \times (10^2)^6 = 64\times 10^{12} = 6.4\times 10^{13}$

3. $2, 6, 10, \cdots, x, y, 26$ is an arithmetic sequence with a common difference of 4.
$y = 26 - 4 = 22, \quad x = y - 4 = 22 - 4 = 18. \quad x + y = 18 + 22 = 40$

4. $3^{-1} + 2^{-2} + 1^{-3} = \frac{1}{3^1} + \frac{1}{2^2} + \frac{1}{1^3} = \frac{1}{3} + \frac{1}{4} + \frac{1}{1} = \frac{4}{12} + \frac{3}{12} + \frac{12}{12} = \frac{19}{12}$

5. The *x*-coordinate is moved 10 – 2 = 8 units and the *y*-coordinate is moved 15 – 5 = 10 units. – 2 + 8 = 6 and - 4 + 10 = 6. The new point is (6, 6).

6. $\sqrt{250} \approx 15.8$ so the first integer that satisfies the statement is 16. $\sqrt{500} \approx 22.4$ so the last integer that satisfies the statement is 22. The integers that satisfy the statement are 16, 17, 18, 19, 20, 21, and 22 (7 integers).

7. 500 – 250 = 250, but that counts the ending number (500). So there are 249 integers. (example: 252 – 250 = 2, but there is only one number, 251, between them)

8. $f(10) = 10^2 + 10 + 17 = 100 + 10 + 17 = 127$
$f(9) = 9^2 + 9 + 17 = 81 + 9 + 17 = 107$
127 -107 = 20

9. $100 - 1.0825\left[2(11.98) + 2(12.94) + 21.50\right] = 100 - 1.0825\left[23.96 + 25.88 + 21.50\right]$
$= 100 - 1.0825(71.34) = 100 - (77.22555) = 100 - 77.23 = 22.77$

10. $4921508 \rightarrow \cancel{4}\,\cancel{9}\,\cancel{2}\,1\,\cancel{5}\,0\,8 \rightarrow 108$

11. $\frac{6 \text{ miles}}{\text{hour}} \times \frac{5280 \text{ feet}}{\text{mile}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = \frac{(6)(5280) \text{ ft}}{(60)(60) \text{ sec}} = 8.8 \frac{\text{ft}}{\text{sec}}$

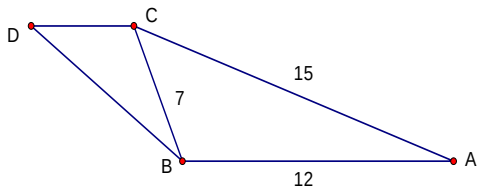
12. Compare the numbers: $2005^{2006} \underline{?}\ 2006^{2005} \rightarrow \frac{2005^{2006}}{2005^{2005}} \underline{?}\ \frac{2006^{2005}}{2005^{2005}} \rightarrow 2005 \underline{?} \left(\frac{2006}{2005}\right)^{2005} \rightarrow$
$2005 \underline{?}\ (1.000498753)^{2005} \rightarrow 2005 > 2.717603624$

13. $\left|(a+b)-(b+c)\right| = \left|a-c\right| = \left|24-35\right| = 11$

14. Being divisible by both 2 and 7 is the same as being divisible by 14. The two-digit multiples of 14 are: 14, 28, 42, 56, 70, 84, and 98 – a total of 7 numbers.

15. $\triangle ABC \sim \triangle BCD$ so

$\frac{AB}{BC} = \frac{AC}{BD}, \frac{12}{7} = \frac{15}{BD}, BD = \frac{7 \cdot 15}{12} = \frac{35}{4}$



16. $1+4\left(\frac{1}{2}\right)+6\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)^3+1\left(\frac{1}{2}\right)^4 = 1+2+6\left(\frac{1}{4}\right)+4\left(\frac{1}{8}\right)+1\left(\frac{1}{16}\right) = 1+2+\frac{6}{4}+\frac{4}{8}+\frac{1}{16} =$
$\frac{16}{16}+\frac{32}{16}+\frac{24}{16}+\frac{8}{16}+\frac{1}{16} = \frac{81}{16}$

$3^3 \cdot 9^3 \cdot 27^3 \cdot 81^3 = 3^3 \cdot \left(3^2\right)^3 \cdot \left(3^3\right)^3 \cdot \left(3^4\right)^3 = 3^3 \cdot 3^6 \cdot 3^9 \cdot 3^{12} =$

17. $3^{3+6+9+12} = 3^{30} = 9^x = \left(3^2\right)^x = 3^{2x}$

$3^{30} = 3^{2x} \rightarrow 30 = 2x \rightarrow 15 = x$

18. $16^5 \times 5^{16} = \left(2^4\right)^5 \times 5^{16} = 2^{20} \times 5^{16} = 2^4\left(2^{16} \times 5^{16}\right) = 16 \times 10^{16}$

16×10^{16} is 16 followed by 16 0's, a total of 18 digits.

19. The distance from A to M is 7 – (- 2) = 9. So it is 9 units from M to B or 7 + 9 = 16.

20. It takes 9 4 in by 4 in tiles to cover 1 square foot. The wall is $5 \times 7 = 35$ square feet. $9 \times 35 = 315$.